

MAS317 Linear Algebra II, 2007

Solutions to Coursework 8.

1. $A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$ clearly has row rank 4 and so its normal form is $Alt(4, 2)$. [5 marks]

- $B = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & 0 \end{pmatrix}$ clearly has row rank 2 and so its normal form is $Alt(4, 1)$. [5 marks]

- $C = \begin{pmatrix} 0 & 1 & 2 & 0 \\ -1 & 0 & -1 & -1 \\ -2 & 1 & 0 & -2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$ clearly has row rank less than 4 as $R_1 = R_4$ and the row rank must be even (as alternating) so it is 2 and so its normal form is $Alt(4, 1)$. [10 marks]

B and C are congruent matrices and A is not congruent to them.

[5 marks]

2. Do row and corresp col ops as in (7.5).

$$\begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & -5 \\ -3 & 5 & 0 \end{pmatrix} R_3 + 5R_1 \quad \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & -5 \\ -3 & 0 & 15 \end{pmatrix} C_3 + 5C_1 \quad \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ -3 & 0 & 0 \end{pmatrix}$$

[10 marks]

$$R_3 + 3R_2 \quad \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} C_3 + 3C_2 \quad \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

[10 marks]

$$(-1)R_1, (-1)C_1 \quad \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = Alt(3, 1).$$

[10 marks]

For P do the col ops only to I_3 .

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} C_3 + 5C_1 \quad [5 \text{ marks}]$$

$$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} C_3 + 3C_2 \quad [5 \text{ marks}]$$

$$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} (-1)C_1 \begin{pmatrix} -1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} = P. \quad [5 \text{ marks}]$$

3. **a).** $(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$, so $A + A^T$ is symmetric. [5 marks]

$(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T)$, so $A - A^T$ is skew symmetric. [5 marks]

Also the ii entry in $A - A^T$ is $(A)_{ii} - (A)_{ii} = 0$ and so $A - A^T$ is alternating. [5 marks]

- b).** Put $B = \frac{1}{2}(A + A^T)$ and $C = \frac{1}{2}(A - A^T)$, then from **a).**, B is symmetric and C is alternating, and clearly $B + C = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = A$. [5 marks]

$$A = \begin{pmatrix} 1 & -2 & 5 \\ 3 & 0 & -2 \\ 1 & 1 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 1/2 & 3 \\ 1/2 & 0 & -1/2 \\ 3 & -1/2 & 6 \end{pmatrix} + \begin{pmatrix} 0 & -5/2 & 2 \\ 5/2 & 0 & -3/2 \\ -2 & 3/2 & 0 \end{pmatrix}. \quad [7 \text{ marks}]$$

We cannot divide by 2 if F is of characteristic 2, so b) would not work. [3 marks]